Modelling of Distributed Systems - Tutorial 6

11th July 2018

Exercise 1 **Security and liveness properties in temporal logic**

In the lecture, we have a simple form of linear temporal logic with operator *"always"* $\Box P$, *"eventually"* $\Diamond P$ and *"nexttime"* $\bigcirc P$, as well as the binary operator *"until"* P **until** Q.

We consider a system with the set of actions, $M = \{a, b, c, d\}$, with $(a \neq b \neq c \neq d)$.

Which of the following are security properties and which are liveness properties?

(a) The actions a and b are always sequential.

(b) After every action c immediately occurs action a.

(c) After every action a or b (sometimes) we find an action c or d.

Describe these characteristics in temporal logic.

Exercise 2 **Stream predicates and linear temporal logic**

Below, ϕ and ψ denote any LTL formulae

(a) For each of the following formulae, give an infinite stream which satisfies the formula and another infinite stream that does not satisfy the formulae.

i) $a \wedge \Box(a \Rightarrow ((\bigcirc\neg a) \wedge (\bigcirc\bigcirc\neg a) \wedge (\bigcirc\bigcirc\bigcirc\neg a) \wedge (\bigcirc\bigcirc\bigcirc\bigcirc a)))$

ii) $(\neg b) \wedge (\bigcirc\neg b) \wedge (\bigcirc\bigcirc\neg b) \wedge (\bigcirc\bigcirc\bigcirc\neg b)$

iii) $\Box((a \wedge \neg b) \Rightarrow (\neg c \wedge \bigcirc\bigcirc\bigcirc\bigcirc b))$

(b) Test to see if the following streams satisfy the appropriate formulae

i) $\Box(a \Rightarrow (\bigcirc\neg a \wedge \bigcirc\bigcirc a))$: $\langle s, t, s, t, s, t, \ldots\rangle$, where $a(s)$ and $\neg a(t)$.

ii) $\Box(a \Rightarrow \Diamond b)$: $\langle u, t, s\rangle ^\frown s^\infty$, where $\neg a(u)$, $a(t)$ and $\neg b(t)$, $\neg b(s)$.

iii) $x \leq y$ **until** $y < x$: $\left\langle \binom{1}{1}, \binom{2}{1}, \binom{3}{7}, \binom{4}{4}, \binom{0}{5}, \binom{2}{2}, \binom{1}{1}, \binom{0}{0}, \binom{3}{3} \right\rangle ^\frown \binom{0}{0}^\infty$, where we consider for this part that the set of states is $\mathbb{N}^2$, x represents the first (upper) component and y the second (lower) component of a state.

(c) Which of these formulae describe the following statement?

„If ψ is true at some point of time, then that time is after $\neg\phi \wedge \neg\psi$ is true, and strictly before this time, ψ is never true."

☐ ... $\neg(\psi$ **until** $\phi)$
☐ ... $\neg(\phi$ **until** $\psi)$
☐ ... $(\neg\psi$ **until** $(\neg\phi \wedge \neg\psi)) \vee \Box\neg\psi$