Modelling of Distributed Systems - Tutorial 3

17th May, 2018

Exercise 1 **Streams, stream operators and stream processing functions**

We start with the concepts, *Stream* and *Stream processing function* and their syntactic and semantic correctness.

Let α be a type with set $M = \mathbb{N} \cup \{\bot\}$, eg. Messages, where $\bot$ symbolizes an undefined result of an action or calculation. Let *Stream* α be a type of *Stream* over α.

We consider the following characteristic functions for *Stream* α:

$$\begin{aligned} \langle\rangle &: \textit{Stream } \alpha \\ \_\,\&\,\_ &: \alpha \times \textit{Stream } \alpha \rightarrow \textit{Stream } \alpha \\ \_\,\hat{}\,\_ &: \textit{Stream } \alpha \times \textit{Stream } \alpha \rightarrow \textit{Stream } \alpha \\ first &: \textit{Stream } \alpha \rightarrow \alpha \\ rest &: \textit{Stream } \alpha \rightarrow \textit{Stream } \alpha \end{aligned}$$

We associate *Stream* α with the set $M^\omega = (M \setminus \{\bot\})^* \cup (M \setminus \{\bot\})^\infty$, i.e. the *defined finite* (*) and *infinite* ($^\infty$) Streams. We then write $\langle\rangle \in M^\omega$ for the *empty* stream and, eg. $\langle a, b, c\rangle = a \,\&\, b \,\&\, c \,\&\, \langle\rangle$ for a stream with the three elements a, b, c. Further, let $x, y \in M \setminus \{\bot\}$ and $s, t, s_1, s_2 \in M^\omega$ be streams over M with $s, s_1 \neq \langle\rangle$.

For *Stream* α we consider, at least, the following axioms:

$$\begin{aligned} \bot \,\&\, s &= \langle\rangle \\ (x \,\&\, s_1)\hat{}\,s_2 &= x \,\&\, (s_1\hat{}\,s_2) \\ first(x \,\&\, s) &= x \\ rest(x \,\&\, s) &= s \end{aligned}$$

Now, please answer, for the given specification for carrier quantities, if the following are correct:

(a) $\textit{first}(\langle 1, 2, 3\rangle) = 1$

(b) $\textit{rest}(\langle 1, 2, \bot, 3, 4\rangle) = \textit{rest}(\langle 1, 2\rangle)$

(c) $s\hat{}\,t = t\hat{}\,s$

(d) $first(s\hat{}\,t) = first(s)$

(e) $(x\hat{}\,s)\hat{}\,t = x\hat{}\,(s\hat{}\,t)$

(f) $\mathit{first}(\mathit{rest}(\mathit{first}(\langle 1,\ 2,\ 3\rangle))) = \perp$

(g) $\mathit{rest}(\mathit{rest}(\mathit{rest}(\mathrm{x}\&(\mathrm{y}\&\mathrm{s})))) = \mathit{rest}(\mathrm{s})$

Now discuss on the basis of *Stream α stream processing function.*

Extra: Discuss how you can accumulate flows around a concept of time?

Exercise 2 Axiomatic specification stream-processing functions

In this exercise we want to specify stream processing functions by interface specifications. A queue is to store data of type *Data* and for the input character, ∇, return a stored value according to FIFO principle. For example, for an input

$$\langle 1, 2, \nabla, 4, \nabla\rangle$$

the output should be

$$\langle \_\,, \_\,, 1, \_\,, 2\rangle$$

should be given.

Model the component by specifying interface assurances.

(Homework) Model the same component, but the stored values should be return according to LIFO principle. For example, for an input

$$\langle 1, 2, \nabla, 4, \nabla\rangle$$

The output should be

$$\langle \_\,, \_\,, 2, \_\,, 4\rangle$$

Exercise 3 Homework

Consider an *integrator component*, which sums up a stream of integers, returning after each new integer the sum of integers received so far.

(a) Give examples for observations on this component for lengths up to three inputs.

(b) Specify the set of observations, O, on the integrator component, *inductively.*