Modelling of Distributed Systems - Tutorial 2

3rd May 2018

Exercise 1 **Reachability, stable predictates and invariants**

Let $(\Sigma, \Delta, \sigma_0)$ be a *state machine*, with all possible states Σ and the transition function $\Delta : \Sigma \to \mathcal{P}(\Sigma)$. Now we can characterize the *set of reachable states* R_Δ as follows:

$$R_\Delta = \bigcup_{i \in \mathbb{N}_0} \Sigma_i \qquad \text{where} \qquad \Sigma_0 = \{\sigma_0\} \quad \text{und} \quad \Sigma_{i+1} = \{\sigma' \in \Delta(\sigma) \mid \sigma \in \Sigma_i\}$$

Thus, $\Sigma_i \subseteq \Sigma$ is the set of states that are reachable after a maximum of i transitions. Furthermore, an *invariant* is a predicate $p : \Sigma \to \mathbb{B}$ for states, such that $\forall \sigma \in R_\Delta : p(\sigma)$, and a *stable predicate* is a predicate $p : \Sigma \to \mathbb{B}$, such that $\forall \sigma, \sigma' : p(\sigma) \wedge \sigma' \in \Delta(\sigma) \Rightarrow p(\sigma')$.

(a) Based on these assumptions and definitions, show that the following assertion is true:

If a stable predicate holds for all initial states of a state machine, then it is an invariant.

(b) What is the difference between the set of all states, set of reachable states, stable predicates and invariants? Why are invariants and stable predicates interesting?

(c) Find an example of an invariant that is not stable. (Homework)

Exercise 2 **Synchronous processes**

Develop a system of a producer, P, and a consumer, C, with local positive integer variables, `p` and `c` respectively, and a global variable, `x`, for co-ordination of the exchange between two processes.

(a) How would you synchronous this exchange?

(b) Define P and C using *Extended Transition System* (ETS).

(c) Describe the concurrent composition of P and C as a single ETS.

(d) Is the composition modular? Define the ETS of the parallel composition. Are there any incompatabilities with parallel systems? (Homework)